Double Pendulum: The Mathematics of Chaos

The mathematics of chaos is becoming more familiar and many people will have heard of the ‘Butterfly Effect’, used as an illustration of the sensitivity of subsequent weather conditions to very small changes of initial conditions. Generally speaking, in a moving or dynamical system, if a non-linearity is present (that is, a graph of cause and effect would not be a straight line) and the system is sensitive to initial conditions, then its state after a period of time will be impossible to predict. But what does this mean in practice? As far as the weather is concerned, the weather at some future time may be predicted from a knowledge of the conditions of the atmosphere at the start of the prediction. However, in practice, it does not matter how accurate this knowledge of the atmosphere is; as time moves on, the weather will diverge more and more from the forecast until it does not resemble it at all!

Remarkably, however, very simple systems can show exactly the same behaviour. A single pendulum is so predictable that it can be used to measure time, a feature which has been used in clocks for centuries. But simply by adding a second pendulum on to the end of the first, the system becomes chaotic and the resultant motion hypnotically fascinating to watch. This activity is a demonstration of such a pendulum.